5.2 Energy Formulation

5.2.1 Internal Energy

The internal energy function is intended to enforce a shape on the deformable contour and to maintain a constant distance between the points in the contour. Additional terms can be added to influence the motion of the contour.

The internal energy function used herein is defined as follows:

$$\alpha E_{int}(v_i) = cE_{con}(v_i) + bE_{bal}(v_i) \tag{5.3}$$

where $E_{con}(v_i)$ is the *continuity* energy that enforces the shape of the contour and $E_{bal}(v_i)$ is a *balloon* force that causes the contour to grow (balloon) or shrink. c and b provide the relative weighting of the energy terms.

Continuity Energy

In the absence of other influences, the continuity energy term coerces an open deformable contour into a straight line and a closed deformable contour into a circle. The formulation of the continuity energy has been adopted from [29]. The energy term for each element, $e_{jk}(v_i)$, in the matrix, $E_{con}(v_i)$ is defined as follows:

$$e_{jk}(v_i) = \frac{1}{l(V)}\left\|p_{jk}(v_i) - \gamma(v_{i-1} + v_{i+1})\right\|^2 \tag{5.4}$$

where $p_{jk}(v_i)$ is the point in the image that corresponds spatially to energy matrix element $e_{jk}(v_i)$.

$\gamma = 0.5$ for an open contour. In this case, the minimum energy point is the point exactly half way between v_{i-1} and v_{i+1}.

For the case of a closed contour, V is given a modulus of n. Therefore, $v_{n+i} = v_i$. γ is then defined as follows:

$$\gamma = \frac{1}{2\cos\left(\frac{2\pi}{n}\right)} \tag{5.5}$$

Here, the point of minimum energy of $E_{con}(v_i)$ is pushed outward so that V becomes a circle. This behavior is illustrated in Figure 5.2.

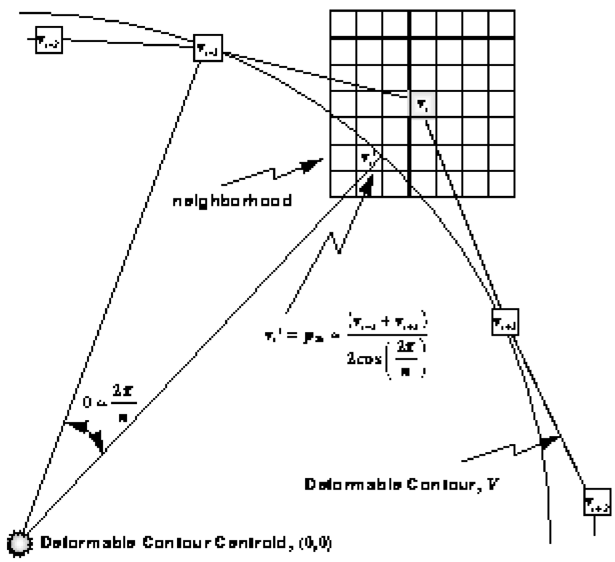

Figure 5.2: An example of the movement of a point in an active contour due to continuity energy. The point, v_i', is the location of minimum energy because it lies on the circle connecting v_{i-1} and v_{i+1}.

The normalization factor, $l(V)$, in Equation 5.4 is the average distance between points in V:

$$l(V) = \frac{1}{n}\sum_{i=1}^{n} \left\| v_{i+1} - v_i \right\|^2 \qquad (5.6)$$

Magnitudes have been left squared to reduce the computational load. The normalization is required to make $E_{con}(v_i)$ independent of the size, location, and orientation of V.

Balloon Force

A balloon force can be used on a closed deformable contour to force the contour to expand (or shrink) in the absence of external influences. A contour initialized within a uniform image object will expand under the influence of a balloon force until it nears the object boundary (at which point the external energy function affects its motion). Figure 5.3 illustrates this behavior.

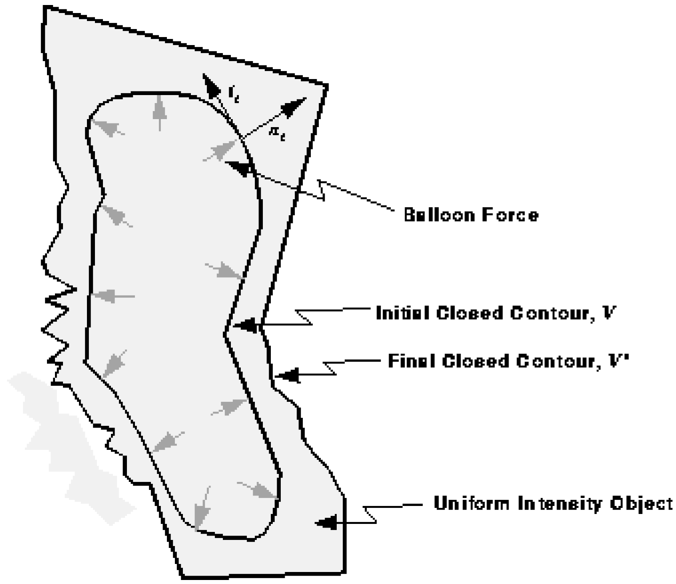

Figure 5.3: An example of the movement of a deformable contour due to balloon energy. Because the object has uniform intensity, a balloon force is required to push the contour toward the object boundary.

Chalana et al. suggest an adaptive balloon force that varies inversely proportionally to the image gradient magnitude [10]. The adaptive balloon force is strong in homogeneous regions and weak near object boundaries, edges, and lines.

The energy term for each element, $c_{jk}(v_i)$, in the matrix, $E_{bal}(v_i)$ is expressed as a dot product:

$$c_{jk}(v_i) = n_i \bullet (v_i - p_{jk}(v_i)) \qquad (5.7)$$

where n_i is the outward unit normal of V at point v_i and $p_{jk}(v_i)$ is the point in the neighborhood of v_i corresponding to entry $c_{jk}(v_i)$ in the energy matrix. Therefore, the balloon energy is smallest at points farthest from v_i in the direction of n_i.

n_i can be found by rotating the tangent vector, t_i, by 90°. t_i is easily computed:

$$t_i = \frac{v_i - v_{i-1}}{\|v_i - v_{i-1}\|} + \frac{v_{i+1} - v_i}{\|v_{i+1} - v_i\|} \qquad (5.8)$$

So n_i is a unit vector normal to t_i.

Adaptive balloon forces are scaled by the image gradient magnitude at point v_i. This can be considered part of the regularization process and is discussed in Section 5.2.3.

5.2.2 External Energy

The external energy function attracts the deformable contour to interesting features, such as object boundaries, in an image. Any energy expression that accomplishes this attraction can be considered for use.

Image gradient and intensity are obvious (and easy) characteristics to look at (another could be object size or shape). Therefore, the following external energy function is investigated:

$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i) \qquad (5.9)$$

where $E_{mag}(v_i)$ is an expression that attracts the contour to high or low intensity regions and $E_{grad}(v_i)$ is an energy term that moves the contour towards edges. Again, the constants, m and g, are provided to adjust the relative weights of the terms.

Image Intensity Energy

Each element in the intensity energy matrix, $E_{mag}(v_i)$ is assigned the intensity value of the corresponding image point in the neighborhood of v_i:

$$e_{jk}(v_i) = I(p_{jk}(v_i)) \qquad (5.10)$$

Then, if m is positive, the contour is attracted to regions of low intensity and vice-versa.

Image Gradient Energy

The image gradient energy function attracts the deformable contour to edges in the image. An energy expression proportional to the gradient magnitude will attract the contour to any edge:

$$e_{jk}(v_i) = -|\nabla I(p_{jk}(v_i))| \qquad (5.11)$$

When active contours are used to find object boundaries, an energy expression that discriminates between edges of adjacent objects is desirable. The key to such an expression is that the gradients at the edges of the objects have different directions. Further, the direction of the gradient at the object's edge should be similar to the direction of the unit normal of the contour. This situation is illustrated in Figure 5.4.

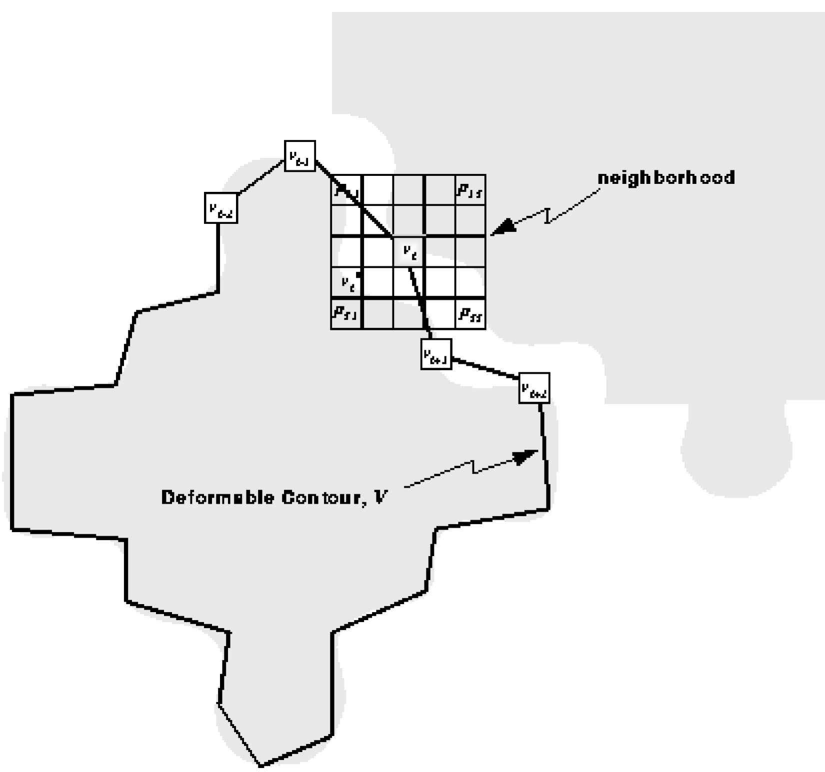

Figure 5.4: An example of the movement of a deformable contour due to gradient energy. Because the gradient direction at the edge of the object of interest is similar to the outward unit normal direction of the contour, the active contour algorithm moves the snake point from $v_i = p_{33}$ to $v_i{}' = p_{41}$ even though the gradient magnitudes at both points are similar.

The value for each element in the directional gradient energy matrix, $E_{grad}(v_i)$, can therefore be defined by a dot product between the unit normal of the deformable contour and the image gradient:

$$c_{jk}(v_i) = -n_i \bullet \nabla I(p_{jk}(v_i)) \tag{5.12}$$

where n_i is the unit normal of the contour at point v_i as defined in Section 5.2.1.

5.2.3 Regularization

The energy functions introduced in the previous sections should be scaled so that the neighborhood matrices contain comparable values. This process is referred to as *regularization*. Here, each of the energy functions is adjusted to the range $[0, 1]$.

The balloon energy is further modified to adapt to the image gradient magnitude. Regularization parameters are added to the intensity and gradient energy terms to stabilize the active contour algorithm.

Continuity Energy

At each point in the deformable contour, the elements in neighborhood matrix for the continuity energy are simply scaled to the range $[0, 1]$:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{c_{max}(v_i) - c_{min}(v_i)} \tag{5.13}$$

where $c_{min}(v_i)$ and $c_{max}(v_i)$ are the minimum and maximum valued elements, respectively, in $E_{con}(v_i)$.

Balloon Energy

The balloon energy is scaled to the range $[0, 1]$, then adapted to the image gradient intensity:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{c_{max}(v_i) - c_{min}(v_i)} \cdot \left(1 - \frac{|\nabla I(v_i)|}{|\nabla I|_{max}}\right) \tag{5.14}$$

where $|\nabla I|_{max}$ is the maximum gradient magnitude in the entire image.

Intensity Energy

A parameter, δI, is added to the intensity energy term for regularization:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{\max(c_{max}(v_i) - c_{min}(v_i), \delta I \cdot I_{max})} \tag{5.15}$$

where I_{max} is the maximum intensity in the entire image and δI has a range of $[0, \infty)$. Therefore, δI determines the sensitivity of the active contour to local variations in image intensity.

Gradient Energy

The gradient energy term is regularized in the same manner as the intensity energy term:

$$e'_{jk}(v_i) = \frac{e_{jk}(v_i) - e_{min}(v_i)}{\max(e_{max}(v_i) - e_{min}(v_i), \delta G \cdot |\nabla I|_{max})} \qquad (5.16)$$

δG also has a range of $[0, \infty)$. A large δG results in an active contour that is insensitive to weak edges.